

Advanced Math Equations and Formulas

Pythagorean Identities:

$$\sin^2 x + \cos^2 x = 1 \quad 1 + \tan^2 x = \sec^2 x \quad \cot^2 x + 1 = \csc^2 x$$

Sum and Difference Formulas:

$$\sin(x \pm y) = \sin x \cos y \pm \sin y \cos x$$

$$\cos(x \pm y) = \cos x \cos y \mp \sin x \sin y$$

$$\tan(x \pm y) = \frac{\tan x \pm \tan y}{1 \mp \tan x \tan y}$$

Double Angle Formulas:

$$\sin 2x = 2 \sin x \cos x$$

$$\begin{aligned} \cos 2x &= \cos^2 x - \sin^2 x \\ &= 2 \cos^2 x - 1 \\ &= 1 - 2 \sin^2 x \end{aligned}$$

$$\tan 2x = \frac{2 \tan x}{1 - \tan^2 x}$$

Half Angle Formulas:

$$\sin \frac{x}{2} = \pm \sqrt{\frac{1 - \cos x}{2}} \quad \cos \frac{x}{2} = \pm \sqrt{\frac{1 + \cos x}{2}} \quad \tan \frac{x}{2} = \frac{1 - \cos x}{\sin x} = \frac{\sin x}{1 + \cos x} = \pm \sqrt{\frac{1 - \cos x}{1 + \cos x}}$$

Power Reducing Formulas:

$$\sin^2 x = \frac{1 - \cos 2x}{2}$$

$$\cos^2 x = \frac{1 + \cos 2x}{2}$$

$$\tan^2 x = \frac{1 - \cos 2x}{1 + \cos 2x}$$

Product to Sum Formulas:

$$\sin x \sin y = \frac{1}{2} [\cos(x - y) - \cos(x + y)] \quad \cos x \cos y = \frac{1}{2} [\cos(x - y) + \cos(x + y)]$$

$$\sin x \cos y = \frac{1}{2} [\sin(x + y) + \sin(x - y)] \quad \cos x \sin y = \frac{1}{2} [\sin(x + y) - \sin(x - y)]$$

Sum to Product Formulas:

$$\sin x + \sin y = 2 \sin \left(\frac{x + y}{2} \right) \cos \left(\frac{x - y}{2} \right) \quad \sin x - \sin y = 2 \cos \left(\frac{x + y}{2} \right) \sin \left(\frac{x - y}{2} \right)$$

$$\cos x + \cos y = 2 \cos \left(\frac{x + y}{2} \right) \cos \left(\frac{x - y}{2} \right) \quad \cos x - \cos y = -2 \sin \left(\frac{x + y}{2} \right) \sin \left(\frac{x - y}{2} \right)$$

Odd and Even Identities:

$$\sin(-x) = -\sin(x)$$

$$\cos(-x) = \cos(x)$$

$$\tan(-x) = -\tan(x)$$

Law of Sines and Cosines:

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc \cos A \quad b^2 = a^2 + c^2 - 2ac \cos B$$

$$c^2 = a^2 + b^2 - 2ab \cos C$$

Triangle Area Formulas:

$$\sqrt{s(s-a)(s-b)(s-c)}$$

$$s = \frac{1}{2}(a+b+c)$$

$$\frac{1}{2}ab \sin C$$

$$\frac{1}{2}bc \sin A$$

$$\frac{1}{2}ac \sin B$$

$$\frac{1}{2}c^2 \frac{\sin A \sin B}{\sin C}$$

$$\frac{1}{2}b^2 \frac{\sin A \sin C}{\sin B}$$

$$\frac{1}{2}a^2 \frac{\sin B \sin C}{\sin A}$$

Sum and Difference of Cubes:

$$a^3 + b^3 = (a+b)(a^2 - ab + b^2)$$

$$a^3 - b^3 = (a-b)(a^2 + ab + b^2)$$

Properties of Logarithms:

$$\log_b(xy) = \log_b x + \log_b y \quad \log_b\left(\frac{x}{y}\right) = \log_b x - \log_b y \quad \log_b x^a = a \log_b x$$

Exponential Applications:

$$f(x) = ae^{bx}$$

$$A = P\left(1 + \frac{r}{n}\right)^{nt}$$

Sequence and Series Formulas:

$$a_n = a_1 + (n-1)d \quad S_n = \frac{n}{2}(a_1 + a_n) \quad S_n = \frac{a_1(1-r^n)}{1-r} \quad S_\infty = \frac{a_1}{1-r}, |r| < 1$$

$$a_n = a_1 r^{n-1}$$

Binomial Theorem:

$$\sum_{r=0}^n \frac{n!}{(n-r)!r!} x^{n-r} y^r$$

$$\sum_{n=0}^r {}_n C_r x^{n-r} y^r$$

Permutation and Combinations:

$${}_n C_r = \frac{n!}{(n-r)!r!}$$

$${}_n P_r = \frac{n!}{(n-r)!}$$

Derivative of a Function:

$$f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$$

$$f'(x) = \lim_{x \rightarrow a} \frac{f(x) - f(a)}{x - a}$$

Distance:

$$\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

Midpoint:

$$\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$$

Polar/Rectangular Conversions:

$$x = r \cos \theta \quad y = r \sin \theta \quad r = \sqrt{x^2 + y^2} \quad \theta = \arctan \frac{y}{x}, x > 0$$